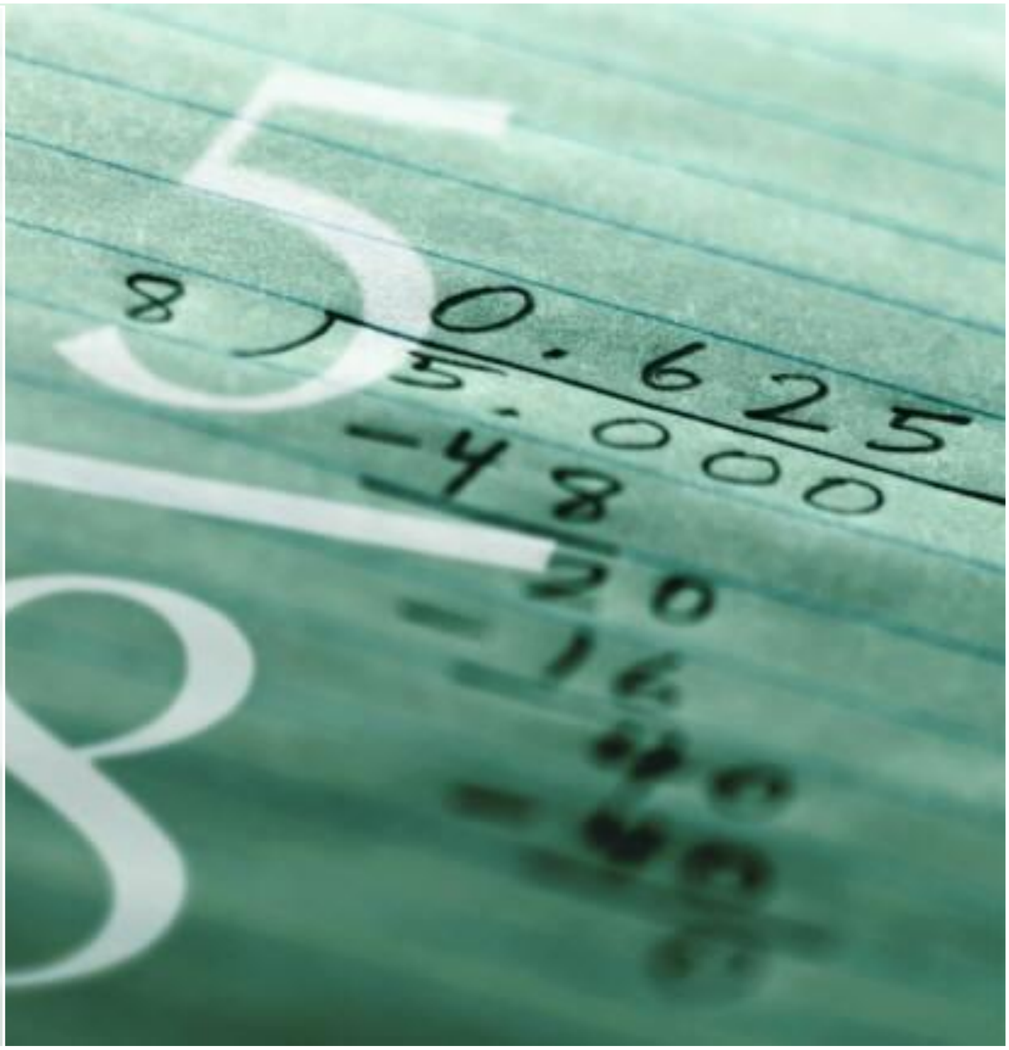
Numerical and Algebraic Fractions

Aquinas Maths Department

Preparation for AS Maths



This unit covers numerical and algebraic fractions. In A level, solutions often involve fractions. This booklet provides a reminder of all the basics and it is best if you don’t use a calculator.

An essential skill in A level is the ability to deal with fractions. In this unit you will do some revision exercises on numerical fractions. Use your GCSE revision guides or perhaps the internet to remind yourself of the rules.

In the second part you will be shown how to deal with algebraic fractions. You will learn how to simplify algebraic fractions and how to add and subtract them.

In order to master the techniques explained here it is vital that you undertake plenty of practice exercises so that all this becomes second nature. To help you to achieve this, the unit includes a substantial number of such exercises. However, you should be using alternative resources in order to get more practice.

After working through this unit, you should be able to:

- perform all four operations on numerical fractions without the use of a calculator (although a calculator is allowed in the A level, the rules for algebraic fractions are the same as those for numerical fractions so it is a good idea to master them with numbers first)
- simplify algebraic fractions
- add and subtract algebraic fractions

Contents

1. Introduction

In this unit you will learn how to manipulate numerical and algebraic fractions.

2. Numerical fractions – a quick review

Exercise 1

1. Express each of the following as a fraction in its simplest form. For example $\frac{3}{21}$ can be written as $\frac{1}{7}$. Remember, no calculators!

a) $\frac{20}{45} =$	e) $\frac{30}{30} =$
b) $\frac{16}{36} =$	f) $\frac{17}{21} =$
c) $-\frac{42}{21} =$	g) $-\frac{49}{35} =$
d) $\frac{18}{16} =$	h) $\frac{90}{30} =$

2. Calculate:

a) $\frac{1}{2}+\frac{1}{3}=$
b) $\frac{1}{2}-\frac{1}{3}=$
c) $\frac{2}{3}+\frac{3}{4}=$
d) $\frac{5}{6}-\frac{2}{3}=$
e) $\frac{8}{9}+\frac{1}{5}+\frac{1}{6}=$
f) $\frac{4}{5}+\frac{3}{7}-\frac{9}{10}=$

3. Evaluate the following, expressing each answer in its simplest form.

a) $\frac{4}{5} \times \frac{3}{16} =$
b) $2 \times 3 \times \frac{1}{4} =$
c) $\frac{3}{4} \times \frac{3}{4} =$
d) $\frac{4}{9} \times 6 =$
e) $\frac{15}{16} \times \frac{4}{5} =$
f) $\frac{9}{5} \times \frac{1}{3} \times \frac{15}{27} =$

4. Evaluate

a) $3 \div \frac{1}{2} =$
b) $\frac{1}{2} \div \frac{1}{4} =$
c) $\frac{6}{7} \div \frac{16}{21} =$
d) $\frac{3/4}{4}$
e) $5 \div \frac{10}{9} =$
f) $\frac{3}{4} \div \frac{4}{3} =$

3. Simplifying Algebraic Fractions

Algebraic fractions have properties which are the same as those for numerical fractions, the only difference being that the numerator (top) and denominator (bottom) are both algebraic expressions.

Example 1

Simplify each of the following fractions.

a) $\frac{2b}{7b^2}$

b) $\frac{3x+x^2}{6x^2}$

Solutions:

a) $\frac{2b}{7b^2} = \frac{2\times b}{7\times b\times b}$

$= \frac{2}{7b}$ Cancel b because it appears in the denominator and the numerator.

b) $\frac{3x+x^2}{6x^2} = \frac{x(3+x)}{x(6x)}$ Factorise first

$= \frac{3+x}{6x}$ Cancel the common factor

Sometimes a little more work is necessary before an algebraic fraction can be simplified.

Example 2

Simplify the algebraic fraction $\frac{x^2-2x+1}{x^2+2x-3}$

Solution:

In this case the numerator and denominator are quadratic expressions which can be factorised first (you should be really good at this now!)

$$\frac{x^2-2x+1}{x^2+2x-3} = \frac{(x-1)^2}{(x-1)(x+3)}$$

$$= \frac{(x-1)(x-1)}{(x-1)(x+3)}$$

$$= \frac{(x-1)}{(x+3)}$$ Cancelling the $(x-1)$

Exercise 2.

Simplify each of the following algebraic fractions.

a) $\frac{8y}{2y^3}$
b) $\frac{2y}{4x}$
c) $\frac{7a^6b^3}{14a^5b^4}$
d) $\frac{(2x)^2}{4x}$
e) $\frac{5y+2y^2}{7y}$

f) $\frac{5ax}{15a+10a^2}$
g) $\frac{2z^2-4z}{2z^2-10z}$
h) $\frac{y^2+7y+10}{y^2-25}$
i) $\frac{w^2-5w-14}{w^2-4w-21}$

So far, simplification has been achieved by cancelling common factors from the numerator and denominator. Sometimes fractions appear in the numerator and/or denominator. In this case you can multiply the numerator and denominator by an appropriate number to obtain an equivalent, simpler expression.

Example 3

Simplify each of the following fractions.

a) $\frac{\frac{1}{4}+y}{\frac{1}{2}}$

b) $\frac{3x+\frac{1}{x}}{2}$

Solutions:

a) $\frac{\frac{1}{4}+y}{\frac{1}{2}} = \frac{4\left(\frac{1}{4}+y\right)}{4\left(\frac{1}{2}\right)}$ To remove the fractions we multiply top and bottom by 4

$= \frac{1+4y}{2}$

b) $\frac{3x+\frac{1}{x}}{2} = \frac{x\left(3x+\frac{1}{x}\right)}{x(2)}$ To simplify multiply numerator and denominator by x

$= \frac{3x^2+1}{2x}$

Exercise 3.

a) $\frac{4y-\frac{3}{2}}{2}$

b) $\dfrac{2x + \frac{1}{2}}{x + \frac{1}{4}}$
c) $\dfrac{z - \frac{1}{3}}{z - \frac{1}{2}}$
d) $\dfrac{2 - \frac{1}{x}}{2}$
e) $\dfrac{3t - \frac{2}{t}}{\frac{1}{2}}$
f) $\dfrac{z - \frac{1}{2z}}{z - \frac{1}{3z}}$

4. Adding Algebraic Fractions

Addition (and subtraction) of algebraic fractions follows exactly the same rules as for numerical fractions.

Example 4

Write the following sum as a single fraction in its simplest form.

$$\frac{2}{x+1} + \frac{1}{x+2}$$

Solution: The *least common multiple* of the denominators is $(x+1)(x+2)$.

$$\frac{2}{x+1} + \frac{1}{x+2} = \frac{2\times(x+2)}{(x+1)\times(x+2)} + \frac{1\times(x+1)}{(x+2)\times(x+1)}$$

$$= \frac{2x+4}{(x+1)(x+2)} + \frac{x+1}{(x+2)(x+1)}$$

$$= \frac{(2x+4)+(x+1)}{(x+2)(x+1)}$$

$$= \frac{3x+5}{(x+2)(x+1)}$$

Exercise 4

a) $\frac{2}{y} + \frac{3}{z}$
b) $\frac{1}{3y} - \frac{2}{5y}$

c) $\frac{3z+1}{3} - \frac{2z+1}{2}$

d) $\frac{3t+1}{2} + \frac{1}{t}$

e) $\frac{x+1}{2} + \frac{1}{x-1}$

f) $\frac{2}{w+3} - \frac{5}{w-1}$

5. Answers

Exercises 1			
1 a) $\frac{4}{9}$	2 a) $\frac{5}{6}$	3 a) $\frac{3}{20}$	4 a) 6
1 b) $\frac{4}{9}$	2 b) $\frac{1}{6}$	3 b) $\frac{3}{2}$	4 b) 2
1 c) -2	2 c) $\frac{17}{12}$	3 c) $\frac{9}{16}$	4 c) $\frac{9}{8}$
1 d) $\frac{9}{8}$	2 d) $\frac{1}{6}$	3 d) $\frac{8}{3}$	4 d) $\frac{3}{16}$
1 e) 1	2 e) $\frac{113}{90}$	3 e) $\frac{3}{4}$	4 e) $\frac{9}{2}$
1 f) $\frac{17}{21}$	2 f) $\frac{23}{70}$	3 f) $\frac{1}{3}$	4 f) $\frac{9}{16}$
1 g) $-\frac{7}{5}$			
1 h) 3			

Exercises 2	Exercises 3	Exercises 4
a) $\frac{4}{y^2}$	a) $\frac{8y-3}{4}$	a) $\frac{3y+2z}{yz}$
b) $\frac{y}{2x}$	b) $\frac{8x+2}{4x+1}$	b) $-\frac{1}{15y}$
c) $\frac{a}{2b}$	c) $\frac{6z-2}{6z-3}$	c) $-\frac{1}{6}$
d) x	d) $\frac{2x-1}{2x}$	d) $\frac{3t^2+t+2}{2t}$
e) $\frac{5+2y}{7}$	e) $\frac{6t^2-4}{t}$	e) $\frac{x^2+1}{2(x-1)}$
f) $\frac{x}{3+2a}$	f) $\frac{6z^2-3}{6z^2-2}$	f) $-\left[\frac{3w+17}{(w+3)(w-1)}\right]$
g) $\frac{z-2}{z-5}$		
h) $\frac{y+2}{y-5}$		
i) $\frac{w+2}{w+3}$		